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Tidal characteristics of Maputo Bay, Mozambique

Sinibaldo Canhanga a, João Miguel Dias b,*

a Department of Oceanography, National Institute of Hydrography and Navigation (INAHINA), Av. Karl Marx, 153 C.P. 2089, Maputo, Mozambique
b Physics Department, University of Aveiro, Campus de Santiago, 3810-193, Aveiro, Portugal

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Abstract

The tidal characteristics of Maputo Bay (a bay located in South part of Mozambique) were assessed in this work through the implementation of a numerical model (SIMSYS2D) and exploration of its numerical results, and by the analysis of observed time series of free surface elevations in Maputo Harbor.

The calibration of the numerical model was carried out based on time series of tidal currents and free surface elevation, which were collected at Maputo Harbor, Baixo Ribeiro and Portuguese Island.

By means of the model results, important harmonic constants of the tidal heights and currents, as well as the form factor, were computed. These results have revealed that there is a phase delay and an increase in amplitude of the major constituents as the tide propagates to the inshore zone. Based on these results, the tidal ellipses in whole Maputo Bay were also computed, which showed the pattern of the tidal currents. The hydrodynamics of the Maputo Bay under extreme tidal conditions were also analyzed (during the largest spring tide and smallest neap tide).

The phase difference between tidal heights and currents revealed that there are no maximum fluxes of energy in most of Maputo Bay and that the mean tidal current (residual) may be different from zero in this system.

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Keywords: Tidal waves; Tidal currents; Harmonic analysis; Form factor; Calibration; Mathematical models; Maputo Bay

1. Introduction

Studies carried out by Prandle (1982), have emphasized that a knowledge of tidal heights and tidal currents structure is required to understand problems such as dispersion, rate of pollutants and sediment transport in coastal areas. Tidal currents also dominate the erosion processes taking place on daily time scales and thereby affect the nutrients budgets (Kitheka et al., 1996). Tidal asymmetries and residual circulation have an influence on nutrient balances, sediment loads, particles and pollutants transportations, etc. (Aldridge, 1997).

Accordingly, the comprehension of the main processes which are ruled by the tidal wave seemed to be important to obtain an overview concerning to the different uses of the bays. Therefore, it is recognized that a basic knowledge of tidal heights and tidal currents is a prerequisite to understand the different processes that are expected to occur in coastal zones, and in this particular study, in the Maputo Bay.

This coastal system is one of the most important in south part of Mozambique (Fig. 1), but numerical
modelling studies or physical observational surveys about this area are rare and non systematic, and therefore its tidal characteristics are barely known. Consequently, it is of interest to perform a study of tides in Maputo Bay.

This aim was achieved by the implementation and application of a numerical model (SYMSYS2D), and by the analysis of its results and of observed time series of free surface elevations in Maputo Harbor.

This work presents the numerical model adopted and describes its calibration. The model was used to determine the distribution of tidal heights and currents, by computing the amplitudes and the phases of the major constituents, the form factor and the tidal ellipses in all the numerical grid points which belong to the Bay. The patterns of tidal flow for the largest spring tide and smallest neap tide were also assessed. The phase difference between the tidal heights and tidal currents was also determined.

2. The study area

The Maputo Bay (Fig. 1), with an area of approximately 1875 km², is located between the latitudes 25° 55' and 26° 10' South and the longitudes 32° 40' and 32° 55' East (Barata et al., 2001). On western side the Bay adjoins Maputo city and joins the Indian Ocean on Northern side. On its eastern side it is limited by the Machangulo peninsula and Inhaca Island.

The Bay is known for its biodiversity (168 species and 4 endemic genera) (Moçambique, 2000), as well as by the fishing activity in its waters. At the entrance of the Bay, there are dynamic sand banks, which endanger navigation into the Bay. It is well known for the increase of pollution due to the industry of smelting aluminium, located in its margins.

More exhaustive descriptions concerning the physical characteristics of Maputo Bay, namely: water temperature, salinity, precipitation, wind, etc, can be found in the works of Hoguane (1994, 1996), Hoguane and Dove (2000), Hoguane et al. (2002), and in Canhanga (2004). In some cases these studies are concerned either with problems in detailed areas or with specific studies.

Guerreiro et al. (1996) and De Boer et al. (2000) performed interesting and complete studies about the sediments and the tides and tidal currents, respectively, at Inhaca Island, on the southern bay. This area is essentially characterized by tidal flats and by narrow and shallow channels, being Ponta Torres Strait the more important, connecting the Maputo Bay and the Indian Ocean. Tidal currents dominate the erosion processes taking place on daily time scales and thereby affect the grain size distribution (Guerreiro et al., 1996).

According to De Boer et al. (2000), the tidal range at Maputo Harbor is about 3 m at spring and analysis of the measured tides at Ponta Torres revealed a form factor of 0.07, indicating a pure semi-diurnal tide, with a spring and neap tidal range of 2.2 and 0.7 m, respectively. At Ponta Torres the ebb currents are about 15 min longer than the flood currents and the ebb flow has a mean velocity of about 0.17 m/s, whereas the maximum currents measured in the channels were about 0.70 m/s (De Boer et al., 2000).

The publications analyzed show that the Bay is considered a shallow system, with water depths in
most parts less than 10 meters. Exceptionally, in zones which are adjacent to the Indian Ocean, the depths are greater than 15 meters. According to the analysis of vertical profiles of salinity and temperature (Hoguane, 1994; Canhanga, 2004), the Bay can be considered vertically homogeneous.

The climate is a mixture of tropical and subtropical, partly influenced by the SE trade wind, and a northerly monsoon, but also occasionally by strong and cold SW winds or cyclones from the NE. The winter (April to September) is usually cold and dry, while the summer (October to March) is warm and rainy. The mean daily air temperature varies from 17 to 27 °C (Hoguane, 1996), from winter to summer, and the mean annual rainfall is 884 mm (De Boer et al., 2000). The Bay receives fresh water mainly from five rivers: Incomati in the North, Maputo in the South, Umbeluzi, Matola and Tembe, in the West. The total fluvial discharge is approximately 190 m³/s.

### 2.1. Tide and tidal currents characterization based on observed data

In Maputo Harbor there is a permanent tide gauge (Fig. 4). Time series of free surface elevation measured at this gauge were supplied by INAHINA, and analyzed in order to determine the harmonic constants. The time series were observed in 1974, 1984, 1986, 1994 and 1998; the length of all time series was one year and the measurements were made every hour.

The amplitudes and phases of the major tidal constituents were computed by using the T_TIDE package (Pawlowicz et al., 2002). The mean latitude used in T_TIDE program was equal to 25.98° South. The results obtained are presented in Table 1.

Analysis of Table 1 shows that in Maputo Harbor the tide is semi-diurnal; there is a difference of at least one order of magnitude between the amplitudes of semi-diurnal and diurnal constituents. The phases of the

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<tbody>
<tr>
<td>M₂</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
</tr>
<tr>
<td>S₂</td>
<td>0.55</td>
<td>0.55</td>
<td>0.54</td>
<td>0.54</td>
<td>0.54</td>
</tr>
<tr>
<td>N₂</td>
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<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>K₂</td>
<td>0.14</td>
<td>0.15</td>
<td>0.15</td>
<td>0.16</td>
<td>0.15</td>
</tr>
<tr>
<td>O₁</td>
<td>0.03</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>K₁</td>
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<td>0.04</td>
<td>0.03</td>
<td>0.06</td>
<td>0.03</td>
</tr>
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Fig. 2. Observed time series of free surface elevation at Maputo Harbor; elevations are in meters and are referred to the hydrographic reference level.
major semi-diurnal constituents ($M_2$ and $S_2$) were approximately $122^\circ$ and $164^\circ$, respectively, corresponding to a delay of about 4 h on the propagation of the tidal wave from the meridian of Greenwich.

In Fig. 2 is represented the sea surface elevation time series measured between 28/4/1993 and 13/5/1993, where the amplitude is in meters and the time in days. The analysis of this figure reveals that there is a strong fortnightly tidal modulation, showing the importance of the neap tide/spring tide cycle in the tidal dynamics of this Bay. The mean spring range is approximately 3.0 meters (maximum and minimum water heights above the reference level of about 4.0 and 1.0 m), whereas the mean neap range is just about 1.0 meters.

Time series of tidal heights in Maputo Harbor, during a period of 10 years (from 1990 to 2000) were predicted using the T_PREDICT package (Pawłowicz et al., 2002), and the harmonic constants previously computed. Thence were determined the tidal amplitudes in extreme tidal cycles, namely in the most extreme spring tide and in the most extreme neap tide. These results are shown on Fig. 3. The maximum range was predicted for the first week of April 1994, and corresponded to about 3.8 meters (maximum and minimum water heights above the reference level of about 4.6 and 0.8 meters); whereas the minimum range in the neap tide was predicted for the third week of September of 1995, and corresponded to 0.2 meters. These results seem to be in accordance with the observed values (Fig. 2), revealing the accuracy on tides prediction in Maputo Harbor by using the T_PREDICT package.

A few studies concerning the tidal currents in Maputo Bay were performed by other authors. The results of the studies carried out by Moura (1973) and Achimo (2000) revealed that there are variations of tidal currents in Maputo Bay from place to place, with maximum velocities between 1.3 and 1.5 m/s during the dry season.

3. The numerical model

There are few observed data in the Bay. Free surface elevation until the present data was only measured at only two ports (Maputo Harbor and Portuguese Island); the length of the time series of free surface elevation is less than a month in Portuguese Island. Regarding tidal currents, there are also only two diamonds in the Bay (Baixo Ribeiro and Portuguese Island), at which the observed time series of the horizontal components of velocity were measured.

The first question arising when the purpose is to investigate the tide and tidal current in Maputo Bay is that it is necessary to augment considerably the spatial and temporal information concerning these variables. How many current meters or tide gauges might be necessary to adequately describe the tide and tidal currents in such a large area? Being aware of the difficulties to perform a study in the whole Maputo Bay, based only on observed data, and considering the advantages of numerical modelling, the development and application of a numerical model seemed to be a good option which could execute this task. Studies of tides in coastal areas, such as the San Francisco Bay (Cheng et al., 1993), Ria de Aveiro (Dias et al., 2000), Shark Bay (Burling et al., 2003) or Great Bay Estuarine System (McLaughlin et al., 2003) are currently carried out based on numerical modelling results.

3.1. Equations

Bearing in mind the characteristics of the Bay (vertically homogeneous) and the process that is intended to be studied (long wave propagation), in this work a vertically integrated bi-dimensional hydrodynamical

Fig. 3. Predicted time series of free surface elevation which corresponds to the cycle of maximum amplitude during the spring tide (left side) and the minimum amplitude during the neap tide (right side).
numerical model (SIMSYS2D) (Leendertse and Gritton, 1971; Leendertse, 1987; Dias, 2001) was considered as adequate to perform this study.

The model solves the second order partial differential equations for depth-averaged fluid flow (shallow water equations) derived from the full three-dimensional Navier–Stokes equations. This gives a system consisting of an equation for conservation of volume and two force momentum equations. The assumptions made are that the fluid should be Newtonian and that the density should be constant. The equations (after the simplifications and assumptions made) are presented below:

\[
\begin{align*}
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} &= fV + g \frac{\partial z}{\partial x} + g \frac{U(U^2 + V^2)^{1/2}}{C^2H} + A_h^2U = 0 \\
\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} &= fU + g \frac{\partial z}{\partial y} + g \frac{V(U^2 + V^2)^{1/2}}{C^2H} + A_h \nabla^2V = 0 \\
\frac{\partial z}{\partial t} + \frac{\partial (HU)}{\partial x} + \frac{\partial (HV)}{\partial y} &= 0
\end{align*}
\]  

(1)

(2)

(3)

where \(A_h\) is the horizontal kinematic eddy viscosity (\(m^2/s\)), \(U\) and \(V\) are the depth-averaged velocity components in \(x\) and \(y\) Cartesian directions, respectively (\(m/s\)), \(H\) is the total depth of water (m), \(g\) is the gravitational acceleration (\(m/s^2\)), \(\zeta\) is the free surface elevation above the local datum (m), \(f\) is the Coriolis parameter (1/s), \(C\) is the Chézy coefficient (\(m^{1/2}/s\)) and \(t\) is the time (s).

The Chézy coefficient depends on the bottom roughness and composition and on the height of the water column and is determined in the present work from the Manning roughness coefficient, \(n\) (\(s/m^{1/3}\)) (Chow, 1959):

\[C = \frac{\sqrt{H}}{n}\]  

(4)

These equations were discretized through the application of the finite difference method, using a space-staggered grid type C of Arakawa. An Alternating Direct Implicit method (ADI) is used to solve these equations (Leendertse and Gritton, 1971; Leendertse, 1987).

With appropriate boundary and initial conditions, this system of equations constitutes a well-posed initial boundary value problem whose solution describes the depth-averaged circulation in a tidal basin. For barotropic models driven by tidal forcing the boundary conditions from experimental data include both the reflected and the incident waves, therefore extrapolation formulae are adopted at open boundaries in this model. A dynamic water elevation at the ocean open boundary was imposed by using pre-determined tidal harmonic constants.

The model treats also shallow water flats in a mass-conserving way. The shallow water flats during a tidal cycle are sometimes dry, and occasionally covered with water. During the dry periods the grid cells representing these areas are taken out of the algebraic system and are later added again, once the adjoining water level is higher than the water inside the dry grid cell. The specific implementation here adopted conserves the mass in each grid cell.

The numerical solution of the model’s equations allow the computation of the velocities and free surface elevation values in all grids points of the numerical domain.

3.2. Model implementation

The knowledge of the Bay’s geometry (the numerical bathymetry) is essential to put the model into operation, because it constitutes a factor which assures the model reality. In order to implement the model, the specification of boundary and initial conditions is also required.

The finite difference equations are solved with the aid of a numerical bathymetry, which was obtained from nautical charts 644, 2930, edited by the Hydrographic Office of UK, and chart 495 edited by Hydrographic Institute of Portugal. The domain (the numerical bathymetry), specifying the bed boundary and the lateral boundaries, comprises a regular grid, spaced by 250×250 m, with 380×350 points on \(y\) and \(x\) directions, respectively (Fig. 4). The domain was rotated 20° eastward from North, relatively to the geographic coordinates, to reduce the grid dimensions in order to improve the computational efficiency.

The eastern side of the domain was considered as an ocean open boundary, where time series of free surface elevation were imposed. These values were computed each 6 min, using T_PREDICT package (Pawlowicz et al., 2002) and the harmonic constants previously determined on Section 2.1. On the closed solid boundaries a free slip condition was assumed and a null value of the normal velocity was imposed, meaning that no mass flux of water takes place on the lateral faces (Koutitas,
1988; Fernandes et al., 2001). The initial conditions were horizontal level and null velocity in all the grid points.

In the implementation of the model were specified the values of 1026 m$^3$/s and 9.81030 m/s$^2$ for the water density and for the gravitational acceleration, respectively. Values of 60 s and 3 days were chosen for the computational time step and for the spin up period, respectively. Sensitivity tests on eddy viscosity were carried out and little difference was observed in the predicted circulation patterns. These results indicate that the eddy viscosity has diminutive effect on the bay circulation, which seems to be mainly controlled by advective processes. A typical value of 20 m$^2$/s was adopted for $A_{bh}$, in order to stabilize the numerical solution with the viscosity.

3.3. Calibration

By definition, a mathematical model is an approximate reconstruction of the processes that occur in reality (Dias, 2001). Obviously, the approximations, simplifications and assumptions which were made, can bring differences between the model results and the observed values. To ensure that the model reproduces accurately the processes which were observed in reality, the simulated values must be compared with the observed ones. In this process some parameters can be adjusted to give a best fit between the model results and the observed values. There is a general lack of set guidelines for model adjustment, however, typically the calibration is accomplished by qualitative comparison between a simulated short time series of free surface elevation or velocity and the observed data for the same location and the same period of time (Cheng et al., 1991). In this study the calibration was carried out by using a set of data measured at Maputo Bay between 28/4–13/5/1993. The ports which were used to calibrate the model are shown in Fig. 4. Free surface elevations were compared for Maputo Harbor and Portuguese Island, whereas components of velocity were compared for Portuguese Island and Baixo Ribeiro.

In the case of this study the calibration of the model has been performed in different ways. First the model has been calibrated qualitatively comparing short time series of predicted values with field data for the available data sets. The root mean square (RMS) errors between the model results and the field data were computed to quantify the model performance. The direct comparison of RMS errors may be adopted to evaluate the model accuracy, but since phase errors and amplitude errors are considered together this method presents some limitations. Therefore, the model was after calibrated against harmonic data consisting in amplitudes and phases of the tide and minor and major semi-axis of tidal currents inside the bay.

A large number of simulations were done in order to improve the adjustment between the simulated and the observed values. This adjustment was improved by changing the bottom friction (through alterations of the Manning coefficient — Eq. (4)) and the open boundary condition. At the ocean open boundary, the model was forced with time series of free surface elevation predicted by the tidal constants for Maputo Harbor. As in this case study the tidal gauge data available to impose the open boundary conditions is located too inland, it was decided to adjust simultaneously the boundary condition and the Manning coefficient. With this approach, the parameterized processes (bottom shear, turbulence, etc) may not be completely well described. Also considering the small number of ports available for the calibration, this practice may be considered as requiring improvements in the future (subject to the availability of new data sets) in order to obtain a more accurate model calibration.

Bearing in mind that the main aim was to make the simulated fit the observed values, in each new simulation, the amplitude and phase of the tidal constituents...
imposed on the open boundary were altered, and simultaneously were adjusted the values of the Manning coefficient. These procedures allow the reduction of the differences between observed and simulated values in both calibration ports. By these processes, the tide was extrapolated to the open boundary by imposing a

<table>
<thead>
<tr>
<th>Constituent</th>
<th>$M_2$</th>
<th>$S_2$</th>
<th>$N_2$</th>
<th>$K_1$</th>
<th>$M_8$</th>
<th>$OQ_1$</th>
<th>$MS_4$</th>
<th>$S_4$</th>
<th>$O_1$</th>
<th>$M_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correction (°)</td>
<td>13.0</td>
<td>-15.7</td>
<td>23.0</td>
<td>-63.1</td>
<td>-41.0</td>
<td>-45.3</td>
<td>151.0</td>
<td>-28.3</td>
<td>17.0</td>
<td>140.0</td>
</tr>
</tbody>
</table>

Fig. 5. Comparison between observed and simulated time series of free surface elevation, and $U$ and $V$ components of velocity.
reduction factor of 0.8 on the tidal amplitude at Maputo Harbor; whereas for adjustment of the phases, there was a phase delay or a phase lead (relative to the constituents in Maputo Harbor), for the major constituents as is shown in Table 2. The ultimate Manning coefficient consists in the establishment of two different sets of values for the computational domain. At the shallow area of Espirito Santo estuary, where the friction must be higher, it corresponded to 0.021 s/m\(^{1/3}\), whereas at the other parts of the Bay it was equal to 0.033 s/m\(^{1/3}\).

In Fig. 5 is shown the comparison between simulated and observed time series after the adjustment previously described. The results of these comparisons show that the measured and modelled curves are nearly coincident, which means that there was a good agreement between these two sets of values (simulated and observed values).

Afterward the (observed and simulated) values of free surface elevation and of velocity components were submitted to a statistical inference. The root mean square error (RMS) between the observed and simulated values was computed. The RMS of tidal currents corresponded to 7.9 and 10.86 cm/s in the Portuguese Island and Baixo Ribeiro, respectively; whereas the RMS of free surface elevation in both calibration ports (Portuguese Island and Maputo Harbor) was 0.013 meters.

The phases and amplitudes of the tidal major harmonic constituents and the minor and major semi-axis...
of tidal ellipses were calculated based on harmonic analysis (Foreman, 1977; Foreman and Henry, 1989; Pawlowicz et al., 2002), for both observed and simulated data. The differences between observed and simulated data were also assessed and are presented in Tables 3 and 4. Discrepancies between the observed and simulated semi-axis (9.69 cm/s) and phase values (3.72°) for $S_2$ constituent in Baixo Ribeiro were higher than all other discrepancies. Despite these $S_2$ tidal current discrepancies, the general results have shown that the numerical model reproduces with a good accuracy the tidal wave propagation in the Bay.

Fig. 6. Tidal ellipses drawn using the observed (dotted line) and simulated (solid line) ellipses parameters for the $M_2$ (on left side) and $S_2$ (on right side) constituents in Baixo Ribeiro and Portuguese Island.
Based on harmonic analysis results a qualitative comparison between the observed and simulated tidal currents was also done (for Baixo Ribeiro and Portuguese Island) by drawing the tidal ellipses determined from their previously determined parameters (Fig. 6). Fig. 6 reveals that there are no substantial discrepancies between the observed and simulated tidal currents for these two diamonds, since the main flow directions and amplitudes are similar in both cases.

The results above mentioned have revealed that the model reproduces correctly the tidal amplitudes and phases, as well as the tidal velocity components, and thus that the model calibration was successfully performed. Therefore, the model satisfactorily reproduces the tidal wave propagation in the Maputo Bay.

4. Tidal characterization of Maputo Bay

The results of the previous section reveal that the amplitude of $K_1$ constituent represents only 5% of the amplitude of the $M_2$ constituent in the ports analyzed. The contribution of the semi-diurnal constituents, $M_2$ and $S_2$, for the total astronomical tide is approximately 90%. This means that the total astronomical tide is well represented by the two major semi-diurnal constituents. The contribution of most other constituents can be considered negligible on the total astronomical tide, suggesting that the constituents with lesser amplitude than the $K_1$ may be neglected on the tidal analysis of the Bay.

Three numerical simulations were carried out in order to establish the tidal characteristics of Maputo Bay. In the first one, the model was forced on the ocean open boundary with a two months time series of tide under mean spring range conditions (Section 2.1). In this simulation the time series of the free surface elevation and of the velocity components were computed at all grid points. The length of these time series was two months, and the time-interval was 1 h. With this data set was determined the amplitude and phase pattern of the main tidal harmonic constituents for the entire bay, as well as the form factor, the tidal ellipses, and the phase difference between the tide and the tidal currents.

The second simulation was performed forcing the model on the ocean open boundary with the tidal cycle corresponding to the highest tide amplitude determined between 1990 and 2000 (Section 2.1). On the third simulation, was imposed on the open boundary the tidal cycle which contained the lowest amplitude between 1990 and 2000 (Section 2.1). These two last simulations allow the assessment of the hydrodynamical behavior of Maputo Bay, under the extreme tidal forcing conditions which are expected to occur in the Bay.

4.1. Amplitudes and phase patterns of the tidal constituents

The Maputo Bay hydrodynamics are ruled by the astronomical tide. Thus, the distribution of phase and amplitude of the main harmonic constituents on the Bay were determined and analyzed. The T_TIDE package (Pawlowicz et al., 2002) was applied to the modelled time series of free surface elevation. For the amplitudes (Fig. 7), there was a variation from the open boundary to the mouth of Espirito Santo Estuary, which spans between 0.65 to 0.95 m (for $M_2$ constituent), and from 0.32 to 0.47 m (for $S_2$ constituent) approximately. The amplitude of the $K_1$ constituent varied from 0.045 to 0.057 m; these values, as expected according to the results of Section 3.3, reveal that the amplitude of the tidal diurnal constituents are one or two orders of magnitude lower than those of the semi-diurnal constituents in whole Maputo Bay.

An increase of the harmonic amplitudes as the wave propagates to the inshore zone was also observed from the analysis of these results, as well as a decrease in the wave amplitude along the main channel (Fig. 7). These results seem to be a consequence of the interaction between the tide and the Bay morphology, and are described by the law of energy conservation. Indeed, for the propagation of the tidal wave in the bay the shallow water theory is generally considered (Pedlosky, 1979). In this theory the velocity of long waves, including tidal waves, is given by (Lewis, 1997):

$$v = \sqrt{gh}$$

where $g$ is the gravitational acceleration (m/s$^2$) and $h$ is the total water height (m).

A more adequate mathematical equation for shallow waters (Doodson and Warburg, 1941) shows that the velocity of propagation is partly dependent upon the surface elevation, and is given by:

$$c = \sqrt{g(h + 3\zeta)}.$$  \hspace{1cm} (6)

As the tidal wave propagates to the inshore zone, there is a decrease of the wave velocity, as a consequence of the decreasing depths. The wave’s kinetic energy (associated with the velocity) can be transformed in potential energy (related with the wave amplitude) or can be dissipated by bed friction (related
with losses of both kinds of mechanical energy) (Lewis, 1997). The results of Fig. 7 show that the effect of bed friction on the central part of the Bay is negligible, when compared with the same effect in the channels.

The results for the phase reveal that it increases when the tidal wave propagates to the inshore zone (Fig. 7); this feature is more relevant in the channels than in the central part of the Bay.

4.2. Form factor

The relative importance of the diurnal and semi-diurnal tidal constituents is sometimes expressed in terms of a form factor, derived from the harmonic constituent amplitudes (Pugh, 2004). In this work, more than the assessment of the amplitudes and the phase patterns, it was also assessed the form factor for the entire Bay (Fig. 8), which was computed based on the harmonic analysis results and on the following expression (Pugh, 1987, 2004):

\[
F = \frac{O_1 + K_1}{M_2 + S_2}
\]  

where \(K_1\), \(O_1\), \(M_2\) and \(S_2\) are the amplitudes (in meters) of the correspondent constituents. From the analysis of Fig. 8 it is verified that the tide in the Bay is essential semi-diurnal, as the form factor is lesser than 0.25 (Pugh, 1987, 2004) in all grid points of the

Fig. 7. Co-tidal charts for the \(M_2\), \(S_2\), \(N_2\) and \(K_1\) harmonic constituents in the Maputo Bay (the amplitudes are expressed in meters and the phases are expressed in degrees).

Fig. 8. Form factor distribution in the Maputo Bay.
computational domain. Examination of this figure has also revealed that the relation between diurnal and semi-diurnal constituents is weaker in the central part of the Bay than in rest of the Bay, as for instance in the channels. These results show that the relationship between the diurnal and semi-diurnal constituents is not constant in the entire Bay and that the semi-diurnal tidal pattern is stronger in central part of the Bay. The results obtained for the area of Inhaca Island are in agreement with the 0.07 value presented by De Boer et al. (2000).

4.3. Tidal ellipses

The determination of tide ellipses for the $M_2$ and $S_2$ constituents allow the assessment of the tidal currents patterns (Fig. 9). In Maputo Bay, the currents attain maximum values (about 1 and 0.5 m/s for the $M_2$ and $S_2$ constituents, respectively) on the central part of the Bay and in the channels; the lower current values were found near to the coast and to the open boundary. The ellipses orientation reveals the importance of the bottom topography and of the bay geometry on the tidal fluxes in Maputo Bay, with the main flux being through the deeper zones.

4.4. Extreme tides

Extreme tidal conditions are presented in this section. Two simulations were performed, where the model was forced on the ocean open boundary with time series corresponding to the tidal cycles containing the extreme amplitudes on spring and neap tide. In order to visualize the evolution of tidal currents and free surface elevation during each of these tidal cycles two animations were performed. Fig. 10 shows the current and the water elevations patterns 3 h after the low water at the open boundary for both spring and neap tide cycles. The time series (the graphics in left superior corner) represents the water elevation imposed on the open boundary; the last asterisk sign (**Q**), corresponds to the instant (on the open boundary) in which the horizontal fields are represented.

Three hours after low water, the currents have attained their maximum values in the central part of the Bay in both situations (spring and neap tide). In this instant, the current intensities lay from 0 to 0.70 m/s and from 0 to 0.34 m/s at spring and neap tides, respectively, whereas the values of free surface elevation reached maximum values of 4.50 and 2.60 m at spring and neap tides, respectively. These results reveal a maximum reduction of approximately 50% of the tidal wave characteristics for the minimum neap tide cycle from the maximum spring tide cycle. There was also a negligible spatial gradient of free surface elevation during the neap tide, whereas during the spring tide there was a discrepancy of almost 1 meter between the wave elevations in the open boundary and in the mouth of the Espirito Santo Estuary. Through the analysis of the animations, it is perceptible that a new flood

Fig. 9. Tidal ellipses distribution in the Maputo Bay ($M_2$ — left side; $S_2$ — right side).
begins 12 h after the establishment of the low tide during the spring tide; however, for the same instant during the neap tide, the animation shows that an ebb current remains on the open boundary. This observation confirms that in the Bay the neap tides propagate slower during the ebb than during the flood and reveals that the semi-diurnal characteristic is much smaller in neap tide.

5. Phase difference between free surface elevation and tidal currents

Due to the action of the bottom friction in shallow waters, advective acceleration and wave drift the free surface elevation and the tidal current can lose their symmetric nature and consequently be distorted. Then the phase difference between the free surface oscillation and tidal current can be different by 90° (Lewis, 1997). It means that the maximum of tidal currents during the rise can occur when the free surface elevation is higher/lower than the mean level. Likewise the maximum currents during the fall may occur when the free surface elevation is lower/higher than the mean level. Consequently, the mean current during the tidal cycle will be weaker/stronger during the fall than during the rise. Thus, measurements of tidal currents in a fixed point can indicate the presence of residual currents when the phase differences between the water surface oscillation and tidal current are not equal to 90°. These residual currents are a matter of detailed analyses (Longuet-Higgins, 1969) and are usually studied to understand the whole dynamics of coastal zones. Another important aspect on the analysis of the phase difference is that the wave mean flux across any transverse section is proportional to the cosine of the phase difference (Lewis, 1997). Thus, the mean flux of energy attains its maximum value when the wave is progressive (the phase difference is zero) and their minimum value on standing waves (the phase difference is 90°). Although the main aim of this work is not to perform an exhaustive study about all these concepts, a concise overview about this subject will help to demonstrate the implications, as well as the importance, in studying and analyzing the phase difference.

Based on the model results, the phase’s difference between the free surface elevation and tidal currents for the main semi-diurnal constituents were assessed. These results are depicted in Fig. 11.

The results show that in the whole domain the tidal wave presents characteristics between those of a progressive and a standing wave, and therefore it can be classified as a mixed wave. In a few areas of the Maputo Bay the phase difference was equal to 0° for both major semi-diurnal constituents, which means that in almost all the area of the Bay the energy flux does not attain its maximum value. As regards the mean tidal current during the tidal cycle, the results reveal an important tidal residual circulation in the bay. Indeed, these results can be a consequence of the weaker cur-
rents occurrences during the fall comparatively to the currents during the rise, or vice-versa.

6. Conclusions

A two dimensional numerical model has been used to examine the tidal dynamics of Maputo Bay. The calibration results have revealed that the model has fairly accurately reproduced the tidal wave propagation in Maputo Bay.

Results of amplitude and form factor have given evidence that in Maputo Bay, the tides are dominated by the major semi-diurnal constituents $M_2$ and $S_2$. There is a phase delay, and an increase of the amplitude of the constituents, as the tidal wave moves toward the inshore zone of Maputo Bay.

From the cycle of minimum tidal amplitude during the neap tide to the cycle of maximum amplitude during the spring tide it was detected a reduction of about 50% in the tidal elevations and current intensities in the Bay. As the wave propagates on the spring tide cycle there was a spatial gradient of free surface elevation of approximately 20 mm/km, between the open boundary and the entrance of Espirito Santo Estuary. The tide on the neap tide cycle seems to be propagated slower during the ebb than during the flood.

The results of phase difference have shown that the tidal wave in Maputo Bay may be classified as a mixed wave. They also revealed that the energy flux is lesser than its maximum possible value in almost all the area of the Bay and that there is evidence of the establishment of residual currents in the Bay. Therefore the residual circulation needs to be carefully determined in future studies.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.jmarsys.2005.08.001.

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